# Solutions for Assignment # 3

by Dru Renner

## Problem 3.1

Let the mass of the particle be $m = 6.0$ kg. The two forces (measured in Newtons) acting on it are

$$\vec{F}_1 = 2\hat{x} - 5\hat{y} + 3\hat{z} \qquad \text{and} \qquad \vec{F}_2 = -4\hat{x} + 8\hat{y} + \hat{z}$$

**(a)** The net force is the vector sum of the two vectors.

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = -2\hat{x} + 3\hat{y} + 4\hat{z}$$

**(b)** The acceleration follows from Newton's Second Law.

$$\vec{a} = \frac{1}{m}\vec{F}_{net} = -\frac{1}{3.0}\hat{x} + \frac{1}{2.0}\hat{y} + \frac{2}{3.0}\hat{z}$$

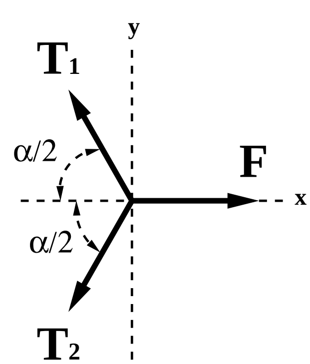
The magnitude of acceleration is given by

$$|a| = \frac{1}{6.0}\sqrt{29} \approx 0.90 \ \text{m/s}^2$$

Note that the mass was given as $m = 6.0$ kg and we were told that the force was given in units of Newtons, so simply "plugging" in the numbers will give an answer in units of $\text{m/s}^2$.

## Problem 3.2

Let $\alpha = 120°$ be the opening angle, $T_1$ be the tension in the upper half of the string, $T_2$ be the tension in the lower half of the string, and $F = 180$ N be the force the archer exerts on the bow string. Note the archer pulls at the middle of the string so the picture below is symmetric.



Equilibrium in the $y$ direction requires

$$T_1 \sin\left(\frac{\alpha}{2}\right) - T_2 \sin\left(\frac{\alpha}{2}\right) = 0 \qquad \Longrightarrow \qquad T_1 = T_2 = T$$

where $T$ is either tension $T_1$ or $T_2$. Equilibrium in the $x$ direction requires

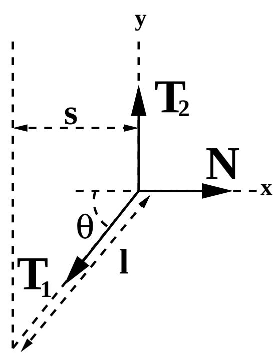
$$F - T\cos\left(\frac{\alpha}{2}\right) - T\cos\left(\frac{\alpha}{2}\right) = 0 \qquad \Longrightarrow \qquad T = \frac{F}{2\cos\left(\frac{\alpha}{2}\right)}$$

So therefore, the tensions in either half of the string are

$$T_1 = T_2 = \frac{180}{2\cos(60^\circ)} = 180 \text{ N}$$

## Problem 3.3

Let $N$, $l$, and $s$ be defined as in the problem and let $T_1$ be the tension in the lower portion of the wire and $T_2$ be the tension in the upper portion of the wire. Assuming that the upper portion of the wire is infinite allows us to draw the upper wire as a straight line.




**(a)** As the picture shows the angle $\theta$ is given by

$$\cos(\theta) = \frac{s}{l}$$

Equilibrium in the $y$ direction requires

$$T_2 - T_1\sin(\theta) = 0 \qquad \Longrightarrow \qquad T_2 = T_1\sin(\theta)$$

Equilibrium in the $x$ direction requires

$$N - T_1\cos(\theta) = 0 \qquad \Longrightarrow \qquad T_1 = \frac{N}{\cos(\theta)}$$

Using the relationship for $\cos(\theta)$ from above gives

$$T_1 = \frac{N}{\frac{s}{l}} = N\left(\frac{l}{s}\right)$$

Using the relationship for $\cos(\theta)$ and the fact that $\sin(\theta) = \sqrt{1 - \cos^2(\theta)}$ gives

$$T_2 = N\left(\frac{l}{s}\right)\sqrt{1 - \left(\frac{s}{l}\right)^2}$$

In the limit of $s \ll l$ ( or $\frac{s}{l} \ll 1$ ) the factor $\sqrt{1 - \left(\frac{s}{l}\right)^2}$ from above becomes 1. So $T_2$ becomes

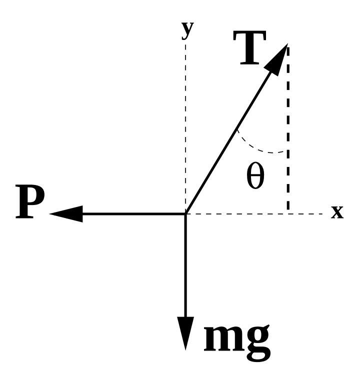
$$T_2 = N\left(\frac{l}{s}\right) = T_1$$

**(b)** Now suppose $s = 2.0$ cm, $l = 1.5$ m, and $N = 150$ N. Then the tension is

$$T = 150\left(\frac{1.5 \times 10^2}{2.0}\right) = 1.1 \times 10^4 \text{ N}$$

## Problem 3.4

Let $m = 2000$ kg, $l = 12$ m, $F = 1800$ N, $T$ be the tension in the wire, and $\theta$ be the angle the cable makes with the vertical.




Equilibrium in the $y$ direction requires

$$T\cos(\theta) - mg = 0 \qquad \Longrightarrow \qquad T = \frac{mg}{\cos(\theta)}$$

Equilibrium in the $x$ direction requires

$$T\sin(\theta) - F = 0 \qquad \Longrightarrow \qquad T = \frac{F}{\sin(\theta)}$$

Combining both equations gives

$$\tan(\theta) = \frac{F}{mg}$$

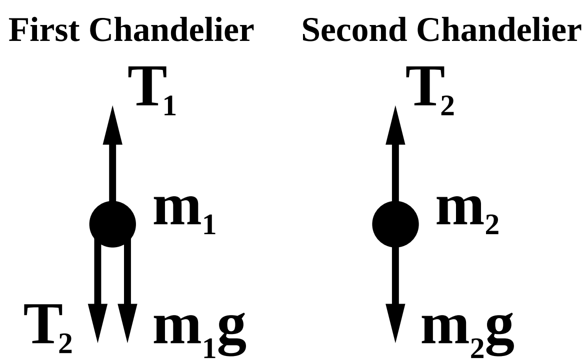
This gives the value of $\theta$ as

$$\theta = \tan^{-1}\left(\frac{F}{mg}\right) = \tan^{-1}\left(\frac{9}{(10)(9.8)}\right) \approx 5.2^\circ$$

Note that the length $l$ was never used.

## Problem 3.5

Let $m_1 = 10$ kg, $m_2 = 3$ kg, $T_1$ be the tension in the first cord, and $T_2$ be the tension in the second cord.




Equilibrium for the first mass $m_1$ requires

$$T_1 - m_1 g - T_2 = 0 \qquad \Longrightarrow \qquad T_1 = m_1 g + T_2$$

Equilibrium for the second mass $m_2$ requires

$$T_2 - m_2 g = 0 \qquad \Longrightarrow \qquad T_2 = m_2 g$$

Combining both equations gives

$$T_2 = m_2 g = 3g \approx 29.4 \text{ N}$$

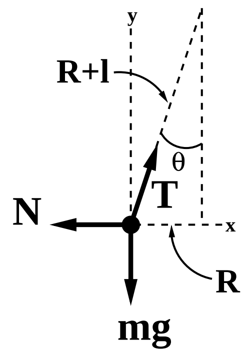
and

$$T_1 = m_1 g + m_2 g = (m_1 + m_2)g = 13g \approx 127.4 \text{ N}$$

Notice that this last answer for $T_1$ is the same answer we would have gotten had we consider both masses $m_1$ and $m_2$ as one object.

## Problem 3.6

Let $m$, $R$, $l$, and $N$ be defined as in the problem and let $T$ be the tension in the cord and $\theta$ be the angle between the cord and the wall.




To do this problem we imagine all the forces acting at the center of the ball. So the shape of the ball is irrelevant except that the length of the cord effectively becomes $R+l$. Trigonometry gives the angle $\theta$ as

$$\sin(\theta) = \frac{R}{R+l}$$

Equilibrium in the $y$ direction requires

$$T\cos(\theta) - mg = 0 \qquad \Longrightarrow \qquad T = \frac{mg}{\cos(\theta)}$$

Equilibrium in the $x$ direction requires

$$T\sin(\theta) - N = 0 \qquad \Longrightarrow \qquad N = T\sin(\theta)$$

Combining both equations gives

$$N = \frac{mg\sin(\theta)}{\cos(\theta)}$$

Using the relationship for $\sin(\theta)$ from above and the fact that $\cos(\theta) = \sqrt{1-\sin^2(\theta)}$ gives

$$\begin{aligned} N &= \frac{mg\sin(\theta)}{\sqrt{1-\sin^2(\theta)}} \\ &= \frac{mg\left(\frac{R}{R+l}\right)}{\sqrt{1-\left(\frac{R}{R+l}\right)^2}} \\ &= \frac{mg}{\sqrt{\left(\frac{R+l}{R}\right)^2-1}} \\ &= \frac{mg}{\sqrt{\left(\frac{l}{R}\right)^2+2\left(\frac{l}{R}\right)}} \end{aligned}$$

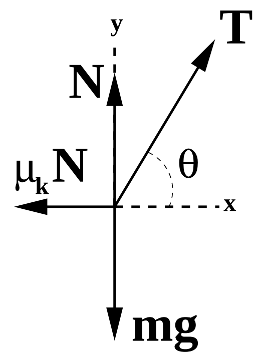
This last form is appropriate for evaluating the limit as $l \to \infty$. You should notice that $l$ only occurs in the combination $\frac{l}{R}$. This quantity is dimensionless, i.e. it has no units. Using a dimensionless quantity helps us figure out what large and small mean for our problem. For this problem, the limit of large $l$ implies that $l$ is many multiples of $R$.

The limit as $l \to \infty$ is clearly

$$N = 0 \text{ for } l \to \infty$$

## Problem 3.7

Let $m$, $\theta$, and $\mu_k$ be as defined and let $T$ be the tension in the rope and $N$ be the normal force on the box.




Equilibrium in the $y$ direction requires

$$N + T\sin(\theta) - mg = 0 \qquad \Longrightarrow \qquad N = mg - T\sin(\theta)$$

Equilibrium in the $x$ direction requires

$$T\cos(\theta) - \mu_k N = 0 \qquad \Longrightarrow \qquad N = \frac{T\cos(\theta)}{\mu_k}$$

Combining both equations gives

$$T = \frac{\mu_k mg}{\cos(\theta) + \mu_k \sin(\theta)}$$

The minimum value for $T$ occurs at $\theta_{\min}$ when

$$\frac{dT}{d\theta} = (-1)\frac{\mu_k mg}{[\cos(\theta_{\min}) + \mu_k \sin(\theta_{\min})]^2}\left[-\sin(\theta_{\min}) + \mu_k \cos(\theta_{\min})\right] = 0$$

which gives an equation for $\theta_{\min}$

$$-\sin(\theta_{\text{min}}) + \mu_k \cos(\theta_{\text{min}}) = 0 \qquad \Longrightarrow \qquad \tan(\theta_{\text{min}}) = \mu_k$$

Using the relationship for $\tan(\theta_{\text{min}})$ from above and the fact that $\cos(\theta) = \sqrt{\frac{1}{1+\tan^2(\theta)}}$ gives

$$\begin{aligned}
T_{\text{min}} &= \frac{\mu_k mg}{\cos\theta_{\text{min}} + \mu_k \sin\theta_{\text{min}}} \\
&= \frac{\mu_k mg}{\cos\theta_{\text{min}}(1 + \mu_k \tan\theta_{\text{min}})} \\
&= \frac{\mu_k mg}{\sqrt{\frac{1}{1+\tan^2(\theta_{\text{min}})}}(1 + \mu_k \tan\theta_{\text{min}})} \\
&= \frac{\mu_k mg}{\sqrt{\frac{1}{1+\mu_k^2}}(1+\mu_k^2)}
\end{aligned}$$

Therefore, the final result for the minimum tension is

$$T_{\text{min}} = \frac{\mu_k mg}{\sqrt{\mu_k^2 + 1}}$$

## Problem 3.8

For the first part of the experiment, Professor Lewin placed a wooden box on an incline and measured the angle at which the box started to slide. So we want to examine the point at which the static force just barely holds the box in place. If we label the angle of the incline relative to the horizontal $\theta$ then the critical angle $\theta_c$ at which the box slides is found by setting the static frictional force to its maximum ($\mathcal{F}_s = \mu_s N$ where N is the normal force) and balancing all forces.

Equilibrium perpendicular to the incline requires

$$N - mg\cos\theta = 0$$

Equilibrium along the incline requires

$$\mathcal{F}_s - mg\sin\theta = 0$$

At the angle $\theta_c$, the box is just about to slide, so the static frictional force is maximum.

$$\mathcal{F}_s = \mu_s N$$

Combining these equations gives the result for $\mu_s$

$$\mu_s = \tan\theta_c$$

The results from the lecture demonstration and the calculated values for $\mu_s$ follow:

| Class | Critical Angle $\theta_c$ | $\mu_s$ |
|---|---|---|
| 10:00 | $20° \pm 2°$ | $0.36 \pm 0.04$ |
| 11:00 | $18° \pm 2°$ | $0.32 \pm 0.04$ |

The calculation of errors follows as:

$$\delta\mu_s = \tan(20° + 2) - \tan 20° \approx 0.04$$

$$\delta\mu_s = \tan(18° + 2) - \tan 18° \approx 0.04$$

For the second part of the experiment, Professor Lewin attached the wooden box, call this $m_1$, to another mass, call this $m_2$, through a pulley. The value of $m_2$ was increased till $m_1$ started to slide up. This corresponds to the moment at which the static friction can no longer prevent motion. This occurs when the force of gravity on $m_1$ directed along the incline ($m_1 g \sin\theta$), the tension ($m_2 g$), and the maximum static friction force ($\mu_s m_1 g \sin\theta$ where $m_1 g \sin\theta$ is the normal force) all balance. For a detailed explanation of this, see problem 3.12. The relevant equation is (3).

$$m_2 g = m_1 g \sin\theta + \mu_s m_1 g \cos\theta$$

The $>$ sign is replaced with an $=$ sign because here the mass $m_1$ is just about to move. Solving the above equation for $\mu_s$ gives

$$\mu_s = \frac{m_2 - m_1 \sin\theta}{m_1 \cos\theta}$$

In lecture the angle $\theta$ was fixed at $\theta = 20° \pm 1°$, the mass $m_1$ was $m_1 = 361 \pm 1g$. The results from the lecture demonstration and the calculated values for $\mu_s$ follow:

| Class | Critical Mass $m_2$ | $\mu_s$ |
|---|---|---|
| 10:00 | $270 \pm 25$ g | $0.43 \pm 0.10$ |
| 11:00 | $245 \pm 15$ g | $0.36 \pm 0.07$ |

The calculation of errors follows as:

$$\delta\mu_s = \frac{(270 + 25) - (361 - 1)\sin(20 - 1)}{(361 - 1)\cos(20 + 1)} - \frac{270 - 361 \sin 20}{361 \cos 20} \approx 0.10$$

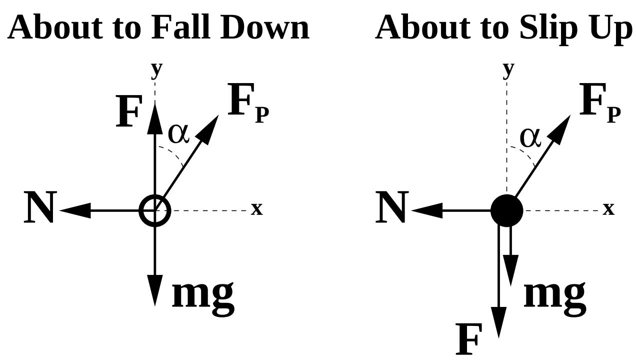
$$\delta\mu_s = \frac{(245 + 15) - (361 - 1)\sin(20 - 1)}{(361 - 1)\cos(20 + 1)} - \frac{270 - 361 \sin 20}{361 \cos 20} \approx 0.07$$

Notice that all the values for $\mu_s$ are consistent with each other!

## Problem 3.9

Let $m$, $\mu$, and $\alpha$ be defined as in the problem and let $F$ be friction. Also let $F_P$ be the force of your push against the book.

**(a)** The free-body diagrams for both cases are




**(b)** First consider the case for which the book is about to fall down. The frictional force points up and is given by

$$F = \mu N$$

Equilibrium in the $y$ direction requires

$$\mu N + F_P \cos(\alpha) - mg = 0$$

Equilibrium in the $x$ direction requires

$$F_P \sin(\alpha) - N = 0 \qquad \Longrightarrow \qquad N = F_P \sin(\alpha)$$

Combining both equations gives

$$F_P = \frac{mg}{\cos(\alpha) + \mu \sin(\alpha)}$$

Now consider the case for which the book is about to slip up. The frictional force points down and is given by

$$F = \mu N$$

Equilibrium in the $y$ direction requires

$$F_P \cos(\alpha) - \mu N - mg = 0$$

Equilibrium in the $x$ direction requires

$$F_P \sin(\alpha) - N = 0 \qquad \Longrightarrow \qquad N = F_P \sin(\alpha)$$

Combining both equations gives

$$F_P = \frac{mg}{\cos(\alpha) - \mu \sin(\alpha)}$$

**(c)** The friction will be zero when there is no relative motion between the wall and the book. This requires that the upward portion of $F_P$ balance the force $mg$, which gives the equation

$$F_P \cos(\alpha) - mg = 0 \qquad \Longrightarrow \qquad F_P = \frac{mg}{\cos(\alpha)}$$

For $\alpha = 0$

$$F_P = \frac{mg}{\cos(0)} = mg$$

For $\alpha = 90°$

$$F_P = \frac{mg}{\cos(90°)} \to \infty$$

**(d)** So lets examine the equation above for the force required to just start the book sliding up.

$$F_P = \frac{mg}{\cos(\alpha) - \mu \sin(\alpha)}$$

As we increase $\mu$ the denominator will become smaller and hence the force $F_P$ will become larger. At some point, call it $\mu^\star$, the denominator becomes zero and the force becomes infinite. This indicates that it is impossible to slide the book up because it would require an infinite push. For $\mu$ larger than $\mu^\star$, the force required to start to slide the book up becomes negative. This is clearly unphysical and indicates that the equation is no longer valid for $\mu$ larger than $\mu^\star$.

$$\cos(\alpha) - \mu^\star \sin(\alpha) = 0 \qquad \Longrightarrow \qquad \mu^\star = \frac{\cos(\alpha)}{\sin(\alpha)} = \frac{1}{\tan(\alpha)}$$

Therefore for $\mu \geq \mu^\star$ it is impossible to slide the book up by applying a push at the angle $\alpha$. We notice that for $\mu < \mu^\star$ the above equation gives a finite value for $F_P$. Thus if we were to push harder than that value the book would indeed slide up. Therefore, for $\mu < \mu^\star$ it is possible to push the book up if we push hard enough.

## Problem 3.10

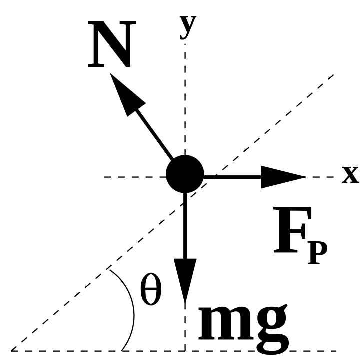
This is the solution from "the Flying Circus of Physics" by Jearl Walker.

The frictional force on the tire does not depend on the surface area in contact with the pavement. Thus a wide slick is as effective as a narrow one. If the tires are spun over the surface as is often done in drag racing, then the wide tries have an advantage in that it has a larger surface area to heat and is less likely to melt. Melting greatly reduces the coefficient of friction.

## Problem 3.11

Let $m = 60$ kg, $\theta = 30°$, $F_P$ be the force with which the woman pushes, and $N$ be the normal force on the box. There is no friction in this problem.

**(a)**




**(b)** Assuming that the box is "at rest or in uniform motion" implies that there is no net force on the box. Then equilibrium in the $y$ direction requires

$$N\cos(\theta) - mg = 0 \qquad \Longrightarrow \qquad N = \frac{mg}{\cos(\theta)}$$

Equilibrium in the $x$ direction requires

$$F_P - N\sin(\theta) = 0 \qquad \Longrightarrow \qquad F_P = N\sin(\theta)$$

Combining both equations gives

$$F_P = mg\tan(\theta)$$

There are three forces acting on the box. The magnitudes are

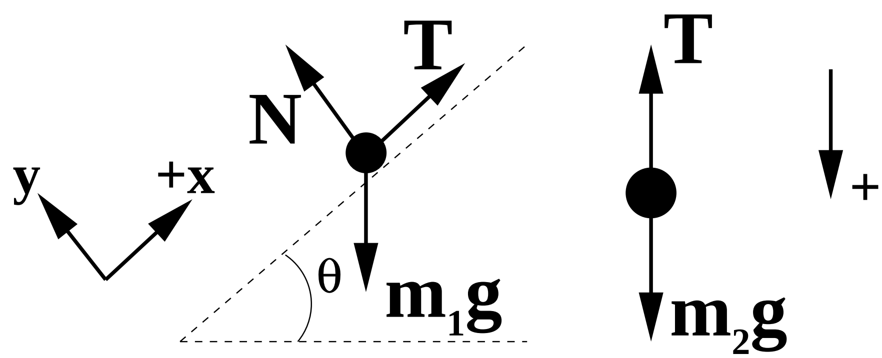
$$mg = 60g \approx 588 \text{ N}$$

$$F_P = mg\tan(\theta) = 60g\tan(30°) \approx 339 \text{ N}$$

$$N = \frac{mg}{\cos(\theta)} = \frac{60g}{\cos(30°)} \approx 679 \text{ N}$$

## Problem 3.12

Let $m_1 = 1.5$ kg, $m_2 = 3.0$ kg, $\theta = 35°$, $\mu_k = 0.40$, $T$ be the tension in the string, and $N$ be the normal force on the mass $m_1$. Let $a$ be the acceleration of either mass.




This problem was worked out in detail in lectures, including a demonstration which allowed you to calculate the static friction coefficient (problem 3.8).

You first have to realize that the tension is the same everywhere in the string. This is ONLY true because: 1) the string is "massless," 2) the pulley is "massless", and 3) it rotates without any friction. In the near future we will deal with pulleys that have a finite mass, and you will see that then the tension on the left of the pulley is different from that on the right.

The forces are shown in the figure. Notice that I did not put the frictional force in (yet). Given the values of $m_2$ and $m_1$, and given the fact that the author ONLY mentions $\mu_k$ (not $\mu_s$), there is little doubt that $m_1$ will be accelerated up-hill. If you assume that, then you can proceed very quickly. You now know that the frictional force is maximum, and that its value is $\mu_k N$, and that the direction is opposite to $T$ (this is the minus x direction in my convention). Since there is no acceleration in the y-direction of $m_1$, you find immediately that

$$N = m_1 g \cos\theta$$

Thus for object 1 we have

$$T - m_1 g \sin\theta - \mu_k m_1 g \cos\theta = m_1 a \tag{1}$$

and for object 2 we have

$$m_2 g - T = m_2 a \tag{2}$$

You can solve for T and for a, and you find

$$a = \frac{g[m_2 - m_1(\sin\theta + \mu_k \cos\theta)]}{m_1 + m_2} \approx 0.37g \approx 3.6 \text{ m/s}^2$$

If you had NO apriori knowledge about the motion, there are three possibilities: 1) $m_1$ is accelerated up-hill, 2) $m_1$ is accelerated down-hill, 3) there is NO acceleration; $m_1$ and $m_2$ stay put. To decide, you now have to know the static friction coefficient. As derived in lectures:

1) will be the case if

$$m_2 g > m_1 g \sin\theta + \mu_s m_1 g \cos\theta \tag{3}$$

2) will be the case if

$$m_2 g < m_1 g \sin\theta - \mu_s m_1 g \cos\theta \tag{4}$$

3) will be the case if neither condition 1) nor condition 2) is met (the frictional force is then in general less than the maximum value possible). It could be directed in the $+x$ direction or the $-x$ direction, it could even be zero. Once you find that e.g. condition 1 is met, you can now proceed. However, you now know that the object is moving, the frictional force is therefore $\mu_k N$, and you proceed with eqs. (1) and (2). Since the author of this problem only gives you the kinetic friction coefficient, he is telling you in an indirect way that the acceleration will not be zero. You can now test for the direction of the acceleration, by using eqs. (3) and (4), replacing $\mu_s$ by $\mu_k$, and you will conclude that indeed $m_1$ is being accelerated up-hill.

## Problem 3.13

Let $N_1$ and $\mathcal{F}_1$ be the normal and frictional forces for one finger, and let $N_2$ and $\mathcal{F}_2$ be the normal and frictional forces for the other finger. The key to understanding this problem is to realize that the fraction of weight supported by each finger can be different. Clearly the finger closest to the center of the yardstick will bear a larger fraction of the weight and hence will exert a larger normal force on the yardstick.

Imagine starting each finger under a separate end of the yardstick. Initially each finger shares the weight equally, but as you attempt to move your fingers one of them, say finger 1, starts to slide. (To avoid sliding you would have to start with your fingers exactly the same distance from each end and move with exactly the same speed. Clearly human fingers are not capable of this. And the yardstick itself is too irregular for that precision.) Immediately after finger 1 slides, both fingers still share the weight equally ($N_1 = N_2$), but because the kinetic coefficient of friction is less than the static coefficient the friction on finger 2 is greater than the friction on finger 1 ($\mathcal{F}_2 = \mu_s N_2 > \mu_k N_1 = \mathcal{F}_1$). As finger 1 continues to slide in, it will bear more of the weight of the yardstick until $N_1$ is large enough that $\mathcal{F}_1 = \mu_k N_1 = \mu_s N_2 = \mathcal{F}_2$. As finger 1 moves in just a bit more, finger 2 will no longer be able to sustain the frictional force from 1, and hence finger 2 will move and finger 1 will stop. The whole procedure will begin again.

## Problem 3.14

Let $k = 150$ N/m, and $L = 0.15$ m. To stretch the spring to twice the relaxed length L requires the force

$$F = k|2L - L| = kL = (150)(0.15) = 22.5 \text{ N}$$

This is really the force per end. If you held both ends then you would have to exert this force for each hand, or if you attached one end to the wall you would have to exert this force for one hand.

To compress the spring to half the relaxed length requires the force

$$F = k|\frac{1}{2}L - L| = \frac{1}{2}kL = \frac{1}{2}(150)(0.15) = 11.25 \text{ N}$$

And again, this is the force per end.

Now let $m = 3$ kg.

**(a)** The angular frequency is given by

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{50} \approx 7.1 \ \frac{\text{rad}}{\text{s}}$$

**(b)** The period of the oscillations is given by

$$\tau = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{50}} \approx 0.89 \text{ s}$$

**(a)** The frequency is given by

$$\nu = \frac{1}{\tau} = \frac{\sqrt{50}}{2\pi} \approx 1.1 \text{ Hz}$$

## Problem 3.15

You aim at the sun. The sun is exactly in the place where it appears to be. It is true that the sun was not on the horizon 8 minutes ago when it emitted the light you are now seeing. (In 8 minutes the Earth rotates about 2°.) However, "the sun wasn't on the horizon" does not mean "the sun moved." It means "the horizon moved." Even refraction of the earth's atmosphere, which will make the sun appear "higher" above the horizon than it really is, does not matter (this will be clear when you take a course on optics).